Mathematica 11.3 Integration Test Results

Test results for the 376 problems in "Stewart Problems.m"

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\,(1-\text{Sin}[x])\,dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\text{Log}[1+\text{Sin}[x]]$$

Result (type 3, 36 leaves):

$$\text{Log}[\text{Cos}[x]]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-\text{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{Cos}[x]}{1-\text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\,\text{Tan}[x]^2\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2}\text{ArcTanh}[\text{Sin}[x]]+\frac{1}{2}\text{Sec}[x]\,\text{Tan}[x]$$

Result (type 3, 42 leaves):

$$\frac{1}{2}\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sec}[x]\,\text{Tan}[x]\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^4\,\text{Csc}[x]^4\,dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\frac{1}{5}\text{Cot}[x]^5-\frac{\text{Cot}[x]^7}{7}$$

Result (type 3, 37 leaves):

$$-\frac{2\,\text{Cot}[x]}{35}-\frac{1}{35}\text{Cot}[x]\,\text{Csc}[x]^2+\frac{8}{35}\text{Cot}[x]\,\text{Csc}[x]^4-\frac{1}{7}\text{Cot}[x]\,\text{Csc}[x]^6$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]\,dx$$

Optimal (type 3, 5 leaves, 1 step):

$$-\text{ArcTanh}[\text{Cos}[x]]$$

Result (type 3, 17 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]^3\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2}\text{ArcTanh}[\text{Cos}[x]]-\frac{1}{2}\text{Cot}[x]\,\text{Csc}[x]$$

Result (type 3, 47 leaves):

$$-\frac{1}{8}\,\text{Csc}\left[\frac{x}{2}\right]^2-\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{8}\,\text{Sec}\left[\frac{x}{2}\right]^2$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[x]\,\text{Cot}[x]\,dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcTanh}[\text{Cos}[x]]+\text{Cos}[x]$$

Result (type 3, 19 leaves):

$$\text{Cos}[x]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 113: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[2x]\,(\text{Cos}[x]+\text{Sin}[x])\,dx$$

Optimal (type 3, 15 leaves, 6 steps):

$$-\frac{1}{2}\,\text{ArcTanh}[\text{Cos}[x]]+\frac{1}{2}\,\text{ArcTanh}[\text{Sin}[x]]$$

Result (type 3, 61 leaves):

$$-\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{-a^2+x^2}}\right]$$

Result (type 3, 46 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{-a^2+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{-a^2+x^2}}\right]$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{a^2+x^2}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{a^2+x^2}}\right]$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-x^2+x^4}\,dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$\frac{1}{x}-\text{ArcTanh}[x]$$

Result (type 3, 22 leaves):

$$\frac{1}{x}+\frac{1}{2}\,\text{Log}[1-x]-\frac{1}{2}\,\text{Log}[1+x]$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]\,(-3+2\,\text{Sin}[x])}{2-3\,\text{Sin}[x]+\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\text{Log}\left[2-3\,\text{Sin}[x]+\text{Sin}[x]^2\right]$$

Result (type 3, 26 leaves):

$$2\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}[2-\text{Sin}[x]]$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2 \,\text{Sin}[x]}{5+\text{Cos}[x]^2}\,dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\sqrt{5}\ \text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{5}}\right]-\text{Cos}[x]$$

Result (type 3, 82 leaves):

$$\frac{1}{20}\left(-\sqrt{5}\ \text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{5}}\right]+21\sqrt{5}\ \text{ArcTan}\left[\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\ \text{Tan}\left[\frac{x}{2}\right]\right]+21\sqrt{5}\ \text{ArcTan}\left[\frac{1}{\sqrt{5}}+\sqrt{\frac{6}{5}}\ \text{Tan}\left[\frac{x}{2}\right]\right]-20\,\text{Cos}[x]\right)$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-4\,\text{Cos}[x]+3\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-\frac{1}{5}\,\text{ArcTanh}\left[\frac{1}{5}\,(3\,\text{Cos}[x]+4\,\text{Sin}[x])\right]$$

Result (type 3, 41 leaves):

$$\frac{1}{5}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-2\,\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{1}{5}\,\text{Log}\left[2\,\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x\sqrt{1+x}}\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-2\,\text{ArcTanh}\left[\sqrt{1+x}\right]$$

Result (type 3, 25 leaves):

$$\text{Log}\left[1-\sqrt{1+x}\right]-\text{Log}\left[1+\sqrt{1+x}\right]$$

Problem 244: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\operatorname{Cos}[x]+\operatorname{Sin}[x]}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[x]-\operatorname{Sin}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 24 leaves):

$$(-1-i)\,(-1)^{3/4}\operatorname{ArcTanh}\left[\frac{-1+\operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]$$

Problem 245: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-\operatorname{Cos}[x]+\operatorname{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$-\operatorname{Log}\left[1+\operatorname{Cot}\left[\frac{x}{2}\right]\right]$$

Result (type 3, 24 leaves):

$$\operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right]-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 246: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{4\operatorname{Cos}[x]+3\operatorname{Sin}[x]}\,dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-\frac{1}{5}\operatorname{ArcTanh}\left[\frac{1}{5}\left(3\operatorname{Cos}[x]-4\operatorname{Sin}[x]\right)\right]$$

Result (type 3, 43 leaves):

$$-\frac{1}{5}\operatorname{Log}\left[2\operatorname{Cos}\left[\frac{x}{2}\right]-\operatorname{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{5}\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+2\operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 249: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]}{1+\text{Sin}[x]} \, dx$$

Optimal (type 3, 18 leaves, 4 steps):

$$\frac{1}{2} \text{ArcTanh}[\text{Sin}[x]] - \frac{1}{2 (1+\text{Sin}[x])}$$

Result (type 3, 54 leaves):

$$\frac{1}{2} \left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \frac{1}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2}\right)$$

Problem 255: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x] \, \text{Tan}[x]^2 \, dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2} \text{ArcTanh}[\text{Sin}[x]] + \frac{1}{2} \text{Sec}[x] \, \text{Tan}[x]$$

Result (type 3, 42 leaves):

$$\frac{1}{2} \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sec}[x] \, \text{Tan}[x]\right)$$

Problem 270: Result more than twice size of optimal antiderivative.

$$\int \left(1+\sqrt{x}\right)^8 dx$$

Optimal (type 2, 27 leaves, 3 steps):

$$-\frac{2}{9} \left(1+\sqrt{x}\right)^9 + \frac{1}{5} \left(1+\sqrt{x}\right)^{10}$$

Result (type 2, 60 leaves):

$$x + \frac{16 x^{3/2}}{3} + 14 x^2 + \frac{112 x^{5/2}}{5} + \frac{70 x^3}{3} + 16 x^{7/2} + 7 x^4 + \frac{16 x^{9/2}}{9} + \frac{x^5}{5}$$

Problem 291: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-e^{-x}+e^{x}}\,dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}\left[e^{x}\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{2}\,\text{Log}\left[1-e^{x}\right]-\frac{1}{2}\,\text{Log}\left[1+e^{x}\right]$$

Problem 297: Result more than twice size of optimal antiderivative.

$$\int (1+\text{Cos}[x])\,\text{Csc}[x]\,dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\text{Log}[1-\text{Cos}[x]]$$

Result (type 3, 20 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}[\text{Sin}[x]]$$

Problem 298: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{x}}{-1+e^{2x}}\,dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}\left[e^{x}\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{2}\,\text{Log}\left[1-e^{x}\right]-\frac{1}{2}\,\text{Log}\left[1+e^{x}\right]$$

Problem 314: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[2x]^{3}\,\text{Csc}[2x]^{3}\,dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{1}{6}\,\mathrm{Csc}[2\,x]^3-\frac{1}{10}\,\mathrm{Csc}[2\,x]^5$$

Result (type 3, 53 leaves):

$$\frac{11\,\mathrm{Cot}[x]}{480}+\frac{11}{960}\,\mathrm{Cot}[x]\,\mathrm{Csc}[x]^2-\frac{1}{320}\,\mathrm{Cot}[x]\,\mathrm{Csc}[x]^4+\frac{11\,\mathrm{Tan}[x]}{480}+\frac{11}{960}\,\mathrm{Sec}[x]^2\,\mathrm{Tan}[x]-\frac{1}{320}\,\mathrm{Sec}[x]^4\,\mathrm{Tan}[x]$$

Problem 328: Result more than twice size of optimal antiderivative.

$$\int x\,\mathrm{Sec}[x]\,\mathrm{Tan}[x]\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\mathrm{ArcTanh}[\mathrm{Sin}[x]]+x\,\mathrm{Sec}[x]$$

Result (type 3, 37 leaves):

$$\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+x\,\mathrm{Sec}[x]$$

Problem 334: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-e^x+e^{3\,x}}\,dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$e^{-x}-\mathrm{ArcTanh}\left[e^x\right]$$

Result (type 3, 32 leaves):

$$e^{-x}+\frac{1}{2}\,\mathrm{Log}\left[1-e^{-x}\right]-\frac{1}{2}\,\mathrm{Log}\left[1+e^{-x}\right]$$

Problem 337: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Sin}[2\,x]}{\sqrt{9-\mathrm{Cos}[x]^4}}\,dx$$

Optimal (type 3, 11 leaves, 5 steps):

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]^2}{3}\right]$$

Result (type 3, 26 leaves):

$$i\,\text{Log}\left[i\,\text{Cos}[x]^2+\sqrt{9-\text{Cos}[x]^4}\right]$$

Problem 351: Result more than twice size of optimal antiderivative.

$$\int e^x\,\text{Sech}\left[e^x\right]\,dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\text{ArcTan}\left[\text{Sinh}\left[e^x\right]\right]$$

Result (type 3, 11 leaves):

$$2\,\text{ArcTan}\left[\text{Tanh}\left[\frac{e^x}{2}\right]\right]$$

Problem 355: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]^5\,dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$\frac{3}{8}\,\text{ArcTanh}[\text{Sin}[x]]+\frac{3}{8}\,\text{Sec}[x]\,\text{Tan}[x]+\frac{1}{4}\,\text{Sec}[x]^3\,\text{Tan}[x]$$

Result (type 3, 58 leaves):

$$\frac{1}{16}\left(-6\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+6\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\,\text{Sec}[x]^4\,(11\,\text{Sin}[x]+3\,\text{Sin}[3\,x])\right)$$

Problem 363: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{5}\,\text{ArcTanh}\left[\frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

Result (type 3, 42 leaves) :

$$-\frac{1}{10}\,\text{Log}\left[1-\frac{x^5}{\sqrt{-2+x^{10}}}\right]+\frac{1}{10}\,\text{Log}\left[1+\frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

Problem 370: Result more than twice size of optimal antiderivative.

$$\int x^2\left(1+x^3\right)^4\,dx$$

Optimal (type 1, 11 leaves, 1 step) :

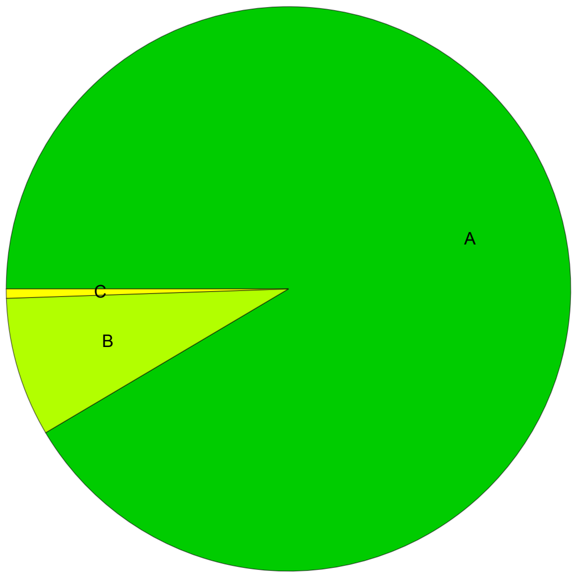
$$\frac{1}{15}\left(1+x^3\right)^5$$

Result (type 1, 36 leaves) :

$$\frac{x^3}{3}+\frac{2\,x^6}{3}+\frac{2\,x^9}{3}+\frac{x^{12}}{3}+\frac{x^{15}}{15}$$

Summary of Integration Test Results

376 integration problems



A - 344 optimal antiderivatives

B - 30 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts